# Math Virtual Learning

## Precalculus with Trigonometry

Students will apply their knowledge of vectors to solve real-world problems.

## May 13th, 2020

Precalculus with Trigonometry
Lesson: May 13th, 2020

**Objective/Learning Target:**
Students will apply their knowledge of vectors to solve real-world problems.

**Let's Get Started!**
**Watch Video: Application of vectors**

# Example: Plane and wind vectors

An airplane is traveling at a speed of 500 miles per hour with a bearing of 330° at a fixed altitude with a negligible wind velocity as shown in Figure 6.32(a). When the airplane reaches a certain point, it encounters a wind with a velocity of 70 miles per hour in the direction N 45° E, as shown in Figure 6.32(b). What are the resultant speed and direction of the airplane?

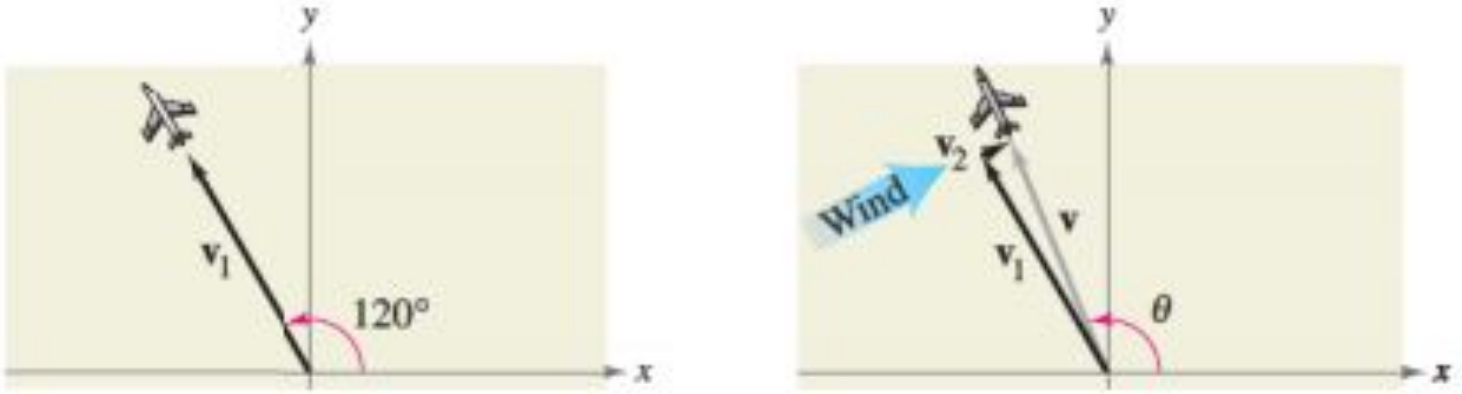



FIGURE 6.32

# Solution:

**First find the component form of both the plane and wind vector by multiplying the magnitude times the cosine and sine of each vector's angle.**

**Then add vectors together and use Pythagorean Theorem to find resultant speed.**

$$\mathbf{v}_1 = 500\langle \cos 120^\circ, \sin 120^\circ \rangle$$
$$= \langle -250, 250\sqrt{3} \rangle$$

and the velocity of the wind is

$$\mathbf{v}_2 = 70\langle \cos 45^\circ, \sin 45^\circ \rangle$$
$$= \langle 35\sqrt{2}, 35\sqrt{2} \rangle.$$

So, the velocity of the airplane (in the wind) is

$$\mathbf{v} = \mathbf{v}_1 + \mathbf{v}_2$$
$$= \langle -250 + 35\sqrt{2}, 250\sqrt{3} + 35\sqrt{2} \rangle$$
$$\approx \langle -200.5, 482.5 \rangle$$

and the resultant speed of the airplane is

$$\|\mathbf{v}\| \approx \sqrt{(-200.5)^2 + (482.5)^2}$$
$$\approx 522.5 \text{ miles per hour.}$$

# Solution continued...

Finally, if $\theta$ is the direction angle of the flight path, you have

$$\tan\theta \approx \frac{482.5}{-200.5}$$

$$\approx -2.4065$$

which implies that

$$\theta \approx 180^\circ + \arctan(-2.4065) \approx 180^\circ - 67.4^\circ = 112.6^\circ.$$

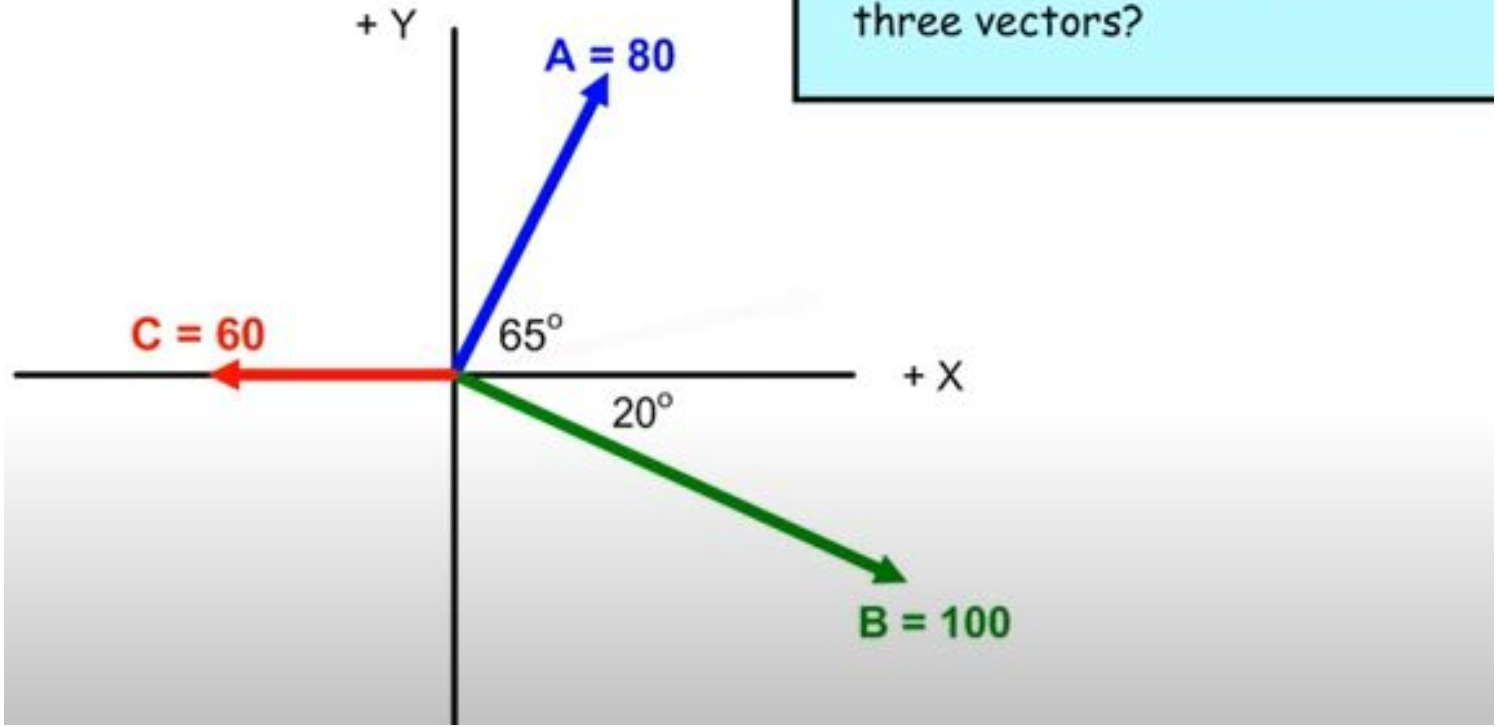
So, the true direction of the airplane is approximately

$$270^\circ + (180^\circ - 112.6^\circ) = 337.4^\circ.$$

# Example: Force vectors

**Vectors A, B, and C pull an object with each force listed.**

What is the resultant of these three vectors?



# Solution to Force Vector

**First, add together the horizontal and vertical components**

| | A | B | C | |
|---|---|---|---|---|
| $\Sigma X =$ | $80\cos 65$ | $+ 100\cos 20$ | $+ {}^{-}60 =$ | |
| $\Sigma X =$ | $33.8$ | $+ 94.0$ | $+ {}^{-}60 =$ | $67.8$ |
| $\Sigma Y =$ | $80\sin 65$ | $+ {}^{-}100\sin 20$ | $+ 0 =$ | |
| $\Sigma Y =$ | $72.5$ | $+ {}^{-}34.2$ | $+ 0 =$ | $38.3$ |

$$R = \sqrt{X^2 + Y^2}$$

$$R = \sqrt{67.8^2 + 38.3^2} = \boxed{77.9}$$

$$\tan\theta = \frac{Y}{X} = \frac{38.3}{67.8}$$

$$\theta = \tan^{-1}\frac{38.3}{67.8} = \boxed{29.5^\circ}$$

**Then use Pythagorean Theorem to find resultant force and inverse tangent to find resultant direction angle.**

# Practice

1 Two tow trucks are pulling on a truck stuck in the mud. Tow truck #1 is pulling with a force of 635 lbs at a 51° from the horizontal while tow truck #2 is pulling with a force of 592 lbs at a 39° from the horizontal. What is the magnitude and direction of the resultant force?

2 A jet is heading due north (which implies to the North) with an airspeed of 500 mph, and the wind is blowing to the southeast at 50 mph. What is the resultant speed of the jet?

3 Lisa and Gina are pulling Aaron in his little red wagon. Gina pulls N 10° E with a force of 160 N (newtons). Lisa pulls N 30° W with a force of 190 N (newtons). What is the magnitude and direction of the resultant?

4 Two forces, one of 35 pounds and the other 50 pounds, act on the same object. The angle between the forces is 30°. Find the magnitude of the resultant vector of these two forces.

# Practice - **ANSWERS**

1. $|\vec{r}| \approx 1220$

   $\theta \approx 45.2°$

2. 465.99 mph

3. 329.1 N, N 11.8° W

4. 82.2 pounds

# Additional Practice and Resources:

Additional Resource Videos:

Tug of War example problem - Khan Academy

Application of vectors examples

Additional Practice:

Application of vectors practice - Khan Academy

Vector application practice